Mexico National Olympiad 1987

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–	Day 1
1	Prove that if the sum of two irreducible fractions is an integer then the two fractions have the same denominator.
2	How many positive divisors does number $20!$ have?
3	Consider two lines ℓ and ℓ' and a fixed point P equidistant from these lines. What is the locus of projections M of P on AB, where A is on ℓ, B on ℓ', and angle $\angle APB$ is right?
4	Calculate the product of all positive integers less than 100 and having exactly three positive divisors. Show that this product is a square.
–	Day 2
5	In a right triangle ABC, M is a point on the hypotenuse BC and P and Q the projections of M on AB and AC respectively. Prove that for no such point M do the triangles BPM, MQC and the rectangle $AQMP$ have the same area.
6	Prove that for every positive integer n the number $(n^3-n)(5^{8n+4}+3^{4n+2})$ is a multiple of 3804.
7	Show that the fraction $\frac{n^2+n-1}{n^2+2n}$ is irreducible for every positive integer n.
8	(a) Three lines l,m,n in space pass through point S. A plane perpendicular to m intersects l,m,n at A,B,C respectively. Suppose that $\angle ASB = \angle BSC = 45^o$ and $\angle ABC = 90^o$. Compute $\angle ASC$. (b) Furthermore, if a plane perpendicular to l intersects l,m,n at P,Q,R respectively and $SP = 1$, find the sides of triangle PQR.