Mexico National Olympiad 1988

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–	Day 1
1	In how many ways can one arrange seven white and five black balls in a line in such a way that there are no two neighboring black balls?
2	If a and b are positive integers, prove that $11a+2b$ is a multiple of 19 if and only if so is $18a+5b$.
3	Two externally tangent circles with different radii are given. Their common tangents form a triangle. Find the area of this triangle in terms of the radii of the two circles.
4	In how many ways can one select eight integers $a_1, a_2, ..., a_8$, not necesarily distinct, such that $1 \le a_1 \le ... \le a_8 \le 8$?
–	Day 2
5	If a and b are coprime positive integers and n an integer, prove that the greatest common divisor of a^2+b^2-nab and $a+b$ divides $n+2$.
6	Consider two fixed points B, C on a circle w. Find the locus of the incenters of all triangles ABC when point A describes w.
7	Two disjoint subsets of the set $\{1, 2, ..., m\}$ have the same sums of elements. Prove that each of the subsets A, B has less than $m/\sqrt{2}$ elements.
8	Compute the volume of a regular octahedron circumscribed about a sphere of radius 1.