Mexico National Olympiad 1989

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–	Day 1
1	In a triangle ABC the area is 18, the length AB is 5, and the medians from A and B are orthogonal. Find the lengths of the sides BC, AC.
2	Find two positive integers a, b such that $a\|b^2, b^2\|a^3, a^3\|b^4, b^4\|a^5$, but a^5 does not divide b^6
3	Prove that there is no 1989-digit natural number at least three of whose digits are equal to 5 and such that the product of its digits equals their sum.
–	Day 2
4	Find the smallest possible natural number $n = \overline{a_m...a_2a_1a_0}$ (in decimal system) such that the number $r = \overline{a_1a_0a_m...{}_20}$ equals $2n$.
5	Let C_1 and C_2 be two tangent unit circles inside a circle C of radius 2. Circle C_3 inside C is tangent to the circles C, C_1, C_2, and circle C_4 inside C is tangent to C, C_1, C_3. Prove that the centers of C, C_1, C_3 and C_4 are vertices of a rectangle.
6	Determine the number of paths from A to B on the picture that go along gridlines only, do not pass through any point twice, and never go upwards? `https://cdn.artofproblemsolving.com/attachments/0/2/87868e24a48a2e130fb5039daeb85af42f4b9` `png`