Mexico National Olympiad 1995

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–	Day 1
1	N students are seated at desks in an $m \times n$ array, where $m, n \geq 3$. Each student shakes hands with the students who are adjacent horizontally, vertically or diagonally. If there are 1020handshakes, what is N?
2	Consider 6 points on a plane such that 8 of the distances between them are equal to 1. Prove that there are at least 3 points that form an equilateral triangle.
3	A, B, C, D are consecutive vertices of a regular 7-gon. AL and AM are tangents to the circle center C radius CB. N is the intersection point of AC and BD. Show that L, M, N are collinear.
–	Day 2
4	Find 26 elements of $\{1, 2, 3, ..., 40\}$ such that the product of two of them is never a square. Show that one cannot find 27 such elements.
5	$ABCDE$ is a convex pentagon such that the triangles ABC, BCD, CDE, DEA and EAB have equal areas. Show that $(1/4)$ area $(ABCDE) <$ area $(ABC) < (1/3)$ area $(ABCDE)$.
6	A 1 or 0 is placed on each square of a 4×4 board. One is allowed to change each symbol in a row, or change each symbol in a column, or change each symbol in a diagonal (there are 14 diagonals of lengths 1 to 4). For which arrangements can one make changes which end up with all 0s?