Mexico National Olympiad 1990

www.artofproblemsolving.com/community/c691189

by parmenides51

–	Day 1

1   How many paths are there from A to the line BC if the path does not go through any vertex twice and always moves to the left?

https://cdn.artofproblemsolving.com/attachments/e/6/a4bc3a9decc06eaeed6f7e99cb58f7b252447
jpg

2 ABC is a triangle with $\angle B = 90^o$ and altitude BH. The inradii of ABC, ABH, CBH are r, r_1, r_2. Find a relation between them.

3   Show that $n^{n-1} - 1$ is divisible by $(n-1)^2$ for $n > 2$.

–	Day 2

4   Find $0/1 + 1/1 + 0/2 + 1/2 + 2/2 + 0/3 + 1/3 + 2/3 + 3/3 + 0/4 + 1/4 + 2/4 + 3/4 + 4/4 + 0/5 + 1/5 + 2/5 + 3/5 + 4/5 + 5/5 + 0/6 + 1/6 + 2/6 + 3/6 + 4/6 + 5/6 + 6/6$

5   Given 19 points in the plane with integer coordinates, no three collinear, show that we can always find three points whose centroid has integer coordinates.

6 ABC is a triangle with $\angle C = 90^o$. E is a point on AC, and F is the midpoint of EC. CH is an altitude. I is the circumcenter of AHE, and G is the midpoint of BC. Show that ABC and IGF are similar.