Mexico National Olympiad 1993

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–	Day 1
1	ABC is a triangle with $\angle A = 90^o$. Take E such that the triangle AEC is outside ABC and $AE = CE$ and $\angle AEC = 90^o$. Similarly, take D so that ADB is outside ABC and similar to AEC. O is the midpoint of BC. Let the lines OD and EC meet at D', and the lines OE and BD meet at E'. Find area $DED'E'$ in terms of the sides of ABC.
2	Find all numbers between 100 and 999 which equal the sum of the cubes of their digits.
3	Given a pentagon of area 1993 and 995 points inside the pentagon, let S be the set containing the vertices of the pentagon and the 995 points. Show that we can find three points of S which form a triangle of area ≤ 1.
–	Day 2
4	$f(n,k)$ is defined by (1) $f(n,0) = f(n,n) = 1$ and (2) $f(n,k) = f(n-1,k-1) + f(n-1,k)$ for $0 < k < n$. How many times do we need to use (2) to find $f(3991,1993)$?
5	OA, OB, OC are three chords of a circle. The circles with diameters OA, OB meet again at Z, the circles with diameters OB, OC meet again at X, and the circles with diameters OC, OA meet again at Y. Show that X, Y, Z are collinear.
6	p is an odd prime. Show that p divides $n(n+1)(n+2)(n+3)+1$ for some integer n iff p divides $m^2 - 5$ for some integer m.