Cono Sur Olympiad 2002

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– Day 1

1 Students in the class of Peter practice the addition and multiplication of integer numbers.The teacher writes the numbers from 1 to 9 on nine cards, one for each number, and places them in an ballot box. Pedro draws three cards, and must calculate the sum and the product of the three corresponding numbers. Ana and Julin do the same, emptying the ballot box. Pedro informs the teacher that he has picked three consecutive numbers whose product is 5 times the sum. Ana informs that she has no prime number, but two consecutive and that the product of these three numbers is 4 times the sum of them. What numbers did Julian remove?

2 Given a triangle ABC, with right $\angle A$, we know: the point T of tangency of the circumference inscribed in ABC with the hypotenuse BC, the point D of intersection of the angle bisector of $\angle B$ with side AC and the point E of intersection of the angle bisector of $\angle C$ with side AB . Describe a construction with ruler and compass for points A, B, and C. Justify.

3 Arnaldo and Bernardo play a Super Naval Battle. Each has a board $n \times n$. Arnaldo puts boats on his board (at least one but not known how many). Each boat occupies the n houses of a line or a column and the boats they can not overlap or have a common side. Bernardo marks m houses (representing shots) on your board. After Bernardo marked the houses, Arnaldo says which of them correspond to positions occupied by ships. Bernardo wins, and then discovers the positions of all Arnaldo's boats. Determine the lowest value of m for which Bernardo can guarantee his victory.

– Day 2

4 Let $ABCD$ be a convex quadrilateral such that your diagonals AC and BD are perpendiculars. Let P be the intersection of AC and BD, let M a midpoint of AB. Prove that the quadrilateral $ABCD$ is cyclic, if and only if, the lines PM and DC are perpendiculars.

5 Consider the set $A=\{1,2,...,n\}$. For each integer k, let r_k be the largest quantity of different elements of A that we can choose so that the difference between two numbers chosen is always different from k. Determine the highest value possible of r_k, where $1 \leq k \leq \frac{n}{2}$

6 Let n a positive integer, $n > 1$. The number n is wonderful if the number is divisible by sum of the your prime factors.
For example; 90 is wondeful, because $90 = 2 \times 3^2 \times 5$ and $2 + 3 + 5 = 10, 10$ divides 90.

Show that, exist a number ”wonderful” with at least 10^{2002} distinct prime numbers.