Vietnam National Olympiad 1979

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–	Day 1
1	Show that for all $x > 1$ there is a triangle with sides, $x^4+x^3+2x^2+x+1, 2x^3+x^2+2x+1, x^4-1$.
2	Find all real numbers a, b, c such that x^3+ax^2+bx+c has three real roots α, β, γ (not necessarily all distinct) and the equation $x^3+\alpha^3x^2+\beta^3x+\gamma^3$ has roots $\alpha^3, \beta^3, \gamma^3$.
3	ABC is a triangle. Find a point X on BC such that : area ABX / area ACX = perimeter ABX / perimeter ACX.
–	Day 2
4	For each integer $n > 0$ show that there is a polynomial $p(x)$ such that $p(2cosx) = 2cosnx$.
5	Find all real numbers k such that $x^2-2x[x]+x-k=0$ has at least two non-negative roots.
6	$ABCD$ is a rectangle with $BC/AB = \sqrt{2}$. $ABEF$ is a congruent rectangle in a different plane. Find the angle DAF such that the lines CA and BF are perpendicular. In this configuration, find two points on the line CA and two points on the line BF so that the four points form a regular tetrahedron.