Vietnam National Olympiad 1978

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by parmenides51

– Day 1

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1 Find all three digit numbers $\overline{abc}$ such that $2 \cdot \overline{abc} = \overline{bca} + \overline{cab}$.

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2 Find all values of the parameter m such that the equations $x^2 = 2^{|x|} + |x| - y - m = 1 - y^2$ have only one root.

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3 The triangle ABC has angle $A = 30^o$ and $AB = \frac{3}{4}AC$. Find the point P inside the triangle which minimizes $5PA + 4PB + 3PC$.

– Day 2

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4 Find three rational numbers $\frac{a}{d}, \frac{b}{d}, \frac{c}{d}$ in their lowest terms such that they form an arithmetic progression and $\frac{b}{a} = \frac{a+1}{d+1}, \frac{c}{b} = \frac{b+1}{d+1}$.

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5 A river has a right-angle bend. Except at the bend, its banks are parallel lines of distance a apart. At the bend the river forms a square with the river flowing in across one side and out across an adjacent side. What is the longest boat of length c and negligible width which can pass through the bend?

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6 Given a rectangular parallelepiped $ABCDA'B'C'D'$ with the bases $ABCD$, $A'B'C'D'$, the edges AA', BB', CC', DD' and $AB = a, AD = b, AA' = c$. Show that there exists a triangle with the sides equal to the distances from A, A', D to the diagonal BD' of the parallelepiped. Denote those distances by m_1, m_2, m_3. Find the relationship between a, b, c, m_1, m_2, m_3.