Spain Mathematical Olympiad 1991

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–	Day 1
1	In the coordinate plane, consider the set of all segments of integer lengths whose endpoints have integer coordinates. Prove that no two of these segments form an angle of 45^o. Are there such segments in coordinate space?
2	Given two distinct elements $a, b \in \{-1, 0, 1\}$, consider the matrix A . Find a subset S of the set of the rows of A, of minimum size, such that every other row of A is a linear combination of the rows in S with integer coefficients.
3	What condition must be satisfied by the coefficients u, v, w if the roots of the polynomial $x^3 - ux^2 + vx - w$ are the sides of a triangle
–	Day 2
4	The incircle of ABC touches the sides BC, CA, AB at A', B', C' respectively. The line $A'C'$ meets the angle bisector of $\angle A$ at D. Find $\angle ADC$.
5	For a positive integer n, let $s(n)$ denote the sum of the binary digits of n. Find the sum $s(1) + s(2) + s(3) + ... + s(2^k)$ for each positive integer k.
6	Find the integer part of $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + ... + \frac{1}{\sqrt{1000}}$