Spain Mathematical Olympiad 1988

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by parmenides51

–	Day 1
1	A sequence of integers $(x_n)_{n=1}^{\infty}$ satisfies $x_1 = 1$ and $x_n < x_{n+1} \leq 2n$ for all $n.$ Show that for every positive integer k there exist indices r, s such that $x_r - x_s = k.$
2	We choose $n > 3$ points on a circle and number them 1 to n in some order. We say that two non-adjacent points A and B are related if, in one of the arcs AB, all the points are marked with numbers less than those at A, B. Show that the number of pairs of related points is exactly $n-3$.
3	Prove that if one of the numbers $25x + 31y, 3x + 7y$ (where $x, y \in Z$) is a multiple of 41, then so is the other.
–	Day 2
4	The Fibonacci sequence is given by $a_1 = 1, a_2 = 2$ and $a_{n+1} = a_n + a_{n-1}$ for $n > 1$. Express a_{2n} in terms of only a_{n-1}, a_n, a_{n+1}.
5	A well-known puzzle asks for a partition of a cross into four parts which are to be reassembled into a square. One solution is exhibited on the picture. `https://cdn.artofproblemsolving.com/attachments/9/1/3b8990baf5e37270c640e280c479b788d989b png` Show that there are infinitely many solutions. (Some solutions split the cross into four equal parts!)
6	For all integral values of parameter t, find all integral solutions (x, y) of the equation $$y^2 = x^4 - 22x^3 + 43x^2 + 858x + t^2 + 10452(t + 39)$$.