**Spain Mathematical Olympiad 1989**

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| – | Day 1 | |
| 1 | An exam at a university consists of one question randomly selected from the$n$ possible questions. A student knows only one question, but he can take the exam $n$ times. Express as a function of $n$ the probability $p_n$ that the student will pass the exam. Does $p_n$ increase or decrease as $n$ increases? Compute $lim_{n\to\infty}p_n$. What is the largest lower bound of the probabilities $p_n$? |
| 2 | Points $A',B',C'$ on the respective sides $BC,CA,AB$ of triangle $ABC$ satisfy $\frac{AC'}{AB}=\frac{BA'}{BC}=\frac{CB'}{CA}=k$. The lines $AA',BB',CC'$ form a triangle $A_1B_1C_1$ (possibly degenerate). Given $k$ and the area $S$ of $\triangle ABC$, compute the area of $\triangle A_1B_1C_1$. |
| 3 | Prove $\frac{1}{10\sqrt2}<\frac{1}{2}\frac{3}{4}\frac{5}{6}...\frac{99}{100}<\frac{1}{10}$ |
| – | Day 2 |
| 4 | Show that the number $1989$ as well as each of its powers $1989^n$ $(n\in N)$, can be expressed as a sum of two positive squares in at least two ways. |
| 5 | Consider the set $D$ of all complex numbers of the form $a+b\sqrt{-13}$ with $a,b\in Z$. The number $14=14+0\sqrt{-13}$ can be written as a product of two elements of $D$: $14=2\cdot7$. Find all possible ways to express $14$ as a product of two elements of $D$. |
| 6 | Prove that among any seven real numbers there exist two,$a$ and $b$, such that $\sqrt3\|a-b\|\le\|1+ab\|$. Give an example of six real numbers not having this property. |