Spain Mathematical Olympiad 1990

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–	Day 1
1	Prove that $\sqrt{x}+\sqrt{y}+\sqrt{xy}$ is equal to$\sqrt{x}+\sqrt{y+xy+2y\sqrt{x}}$ and compare the numbers $\sqrt{3}+\sqrt{10+2\sqrt{3}}$ and $\sqrt{5+\sqrt{22}}+\sqrt{8-\sqrt{22}+2\sqrt{15-3\sqrt{22}}}$.
2	Every point of the plane is painted with one of three colors. Can we always find two points a distance 1 cm apart which are of the same color?
3	Prove that $\lfloor (4+\sqrt{11})^n \rfloor$ is odd for every natural number n.
–	Day 2
4	Prove that the sum $\sqrt[3]{\frac{a+1}{2}+\frac{a+3}{6}\sqrt{\frac{4a+3}{3}}}+\sqrt[3]{\frac{a+1}{2}-\frac{a+3}{6}\sqrt{\frac{4a+3}{3}}}$ is independent of a for $a \geq -\frac{3}{4}$ and evaluate it.
5	On the sides BC, CA and AB of a triangle ABC of area S are taken points A', B', C' respectively such that $AC'/AB = BA'/BC = CB'/CA = p$, where $0 < p < 1$ is variable. (a) Find the area of triangle $A'B'C'$ in terms of p. (b) Find the value of p which minimizes this area. (c) Find the locus of the intersection point P of the lines through A' and C' parallel to AB and AC respectively.
6	There are n points in the plane so that no two pairs are equidistant. Each point is connected to the nearest point by a segment. Show that no point is connected to more than five points.