Spain Mathematical Olympiad 1985

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–	Day 1
1	Let $f : P \to P$ be a bijective map from a plane P to itself such that: (i) $f(r)$ is a line for every line r, (ii) $f(r)$ is parallel to r for every line r. What possible transformations can f be?
2	Determine if there exists a subset E of $Z \times Z$ with the properties: (i) E is closed under addition, (ii) E contains $(0,0)$, (iii) For every $(a,b) \neq (0,0)$, E contains exactly one of (a,b) and $-(a,b)$. Remark: We define $(a,b)+(a',b') = (a+a', b+b')$ and $-(a,b) = (-a,-b)$.
3	Solve the equation $tan^2 2x + 2tan2xtan3x = 1$
4	Prove that for each positive integer k there exists a triple (a,b,c) of positive integers such that $abc = k(a+b+c)$. In all such cases prove that $a^3+b^3+c^3$ is not a prime.
–	Day 2
5	Find the equation of the circle in the complex plane determined by the roots of the equation $z^3+(-1+i)z^2+(1-i)z+i = 0$.
6	Let OX and OY be non-collinear rays. Through a point A on OX, draw two lines r_1 and r_2 that are antiparallel with respect to $\angle XOY$. Let r_1 cut OY at M and r_2 cut OY at N. (Thus, $\angle OAM = \angle ONA$). The bisectors of $\angle AMY$ and $\angle ANY$ meet at P. Determine the location of P.
7	Find the values of p for which the equation $x^5 - px - 1 = 0$ has two roots r and s which are the roots of equation $x^2 - ax + b = 0$ for some integers a, b.
8	A square matrix is sum-magic if the sum of all elements in each row, column and major diagonal is constant. Similarly, a square matrix is product-magic if the product of all elements in each row, column and major diagonal is constant. Determine if there exist 3×3 matrices of real numbers which are both sum-magic and product-magic.