Spain Mathematical Olympiad 1986

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–	Day 1
1	Define the distance between real numbers x and y by $d(x,y) = \sqrt{([x]-[y])^2 + (\{x\}-\{y\})^2}$. Determine (as a union of intervals) the set of real numbers whose distance from $3/2$ is less than $202/100$.
2	A segment d is said to divide a segment s if there is a natural number n such that $s = nd = d + d + ... + d$ (n times). (a) Prove that if a segment d divides segments s and s' with $s < s'$, then it also divides their difference $s' - s$. (b) Prove that no segment divides the side s and the diagonal s' of a regular pentagon (consider the pentagon formed by the diagonals of the given pentagon without explicitly computing the ratios).
3	Find all natural numbers n such that $5^n + 3$ is a power of 2
–	Day 2
4	Denote by $m(a,b)$ the arithmetic mean of positive real numbers a, b. Given a positive real function g having positive derivatives of the first and second order, define $\mu(a,b)$ the mean value of a and b with respect to g by $2g(\mu(a,b)) = g(a) + g(b)$. Decide which of the two mean values m and μ is larger.
5	Consider the curve Γ defined by the equation $y^2 = x^3 + bx + b^2$, where b is a nonzero rational constant. Inscribe in the curve Γ a triangle whose vertices have rational coordinates.
6	Evaluate $$\prod_{k=1}^{14} cos(\frac{k\pi}{15})$$