Mathematical Olympiad 2018

www.artofproblemsolving.com/community/c692750

by cjquines0

1 In triangle ABC with $\angle ABC = 60^\circ$ and $5AB = 4BC$, points D and E are the feet of the altitudes from B and C, respectively. M is the midpoint of BD and the circumcircle of triangle BMC meets line AC again at N. Lines BN and CM meet at P. Prove that $\angle EDP = 90^\circ$.

---

2 Suppose $a_1, a_2, \ldots$ is a sequence of integers, and d is some integer. For all natural numbers n,

(i)$|a_n|$ is prime; (ii)$a_{n+2} = a_{n+1} + a_n + d$.

Show that the sequence is constant.

---

3 Let n be a positive integer. An $n \times n$ matrix (a rectangular array of numbers with n rows and n columns) is said to be a platinum matrix if:

- the n^2 entries are integers from 1 to n;
- each row, each column, and the main diagonal (from the upper left corner to the lower right corner) contains each integer from 1 to n exactly once; and
- there exists a collection of n entries containing each of the numbers from 1 to n, such that no two entries lie on the same row or column, and none of which lie on the main diagonal of the matrix.

Determine all values of n for which there exists an $n \times n$ platinum matrix.

---

4 Determine all ordered pairs (x, y) of nonnegative integers that satisfy the equation

$$3x^2 + 2 \cdot 9^y = x(4^{y+1} - 1).$$
