Argentina Cono Sur Team Selection Test 2012

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by Leicich

–	**Day 1**

1   Sofa colours 46 cells of a 9×9 board red. If Pedro can find a 2×2 square from the board that has 3 or more red cells, he wins; otherwise, Sofa wins. Determine the player with the winning strategy.

2   Find all four-element sets of positive integers $\{w, x, y, z\}$ such that $w^x + w^y = w^z$.

3 16 people sit around a circular table. After some time, they all stand up and sit down in either the chair they were previously sitting on or on a chair next to it. Determine the number of ways that this can be done.

    Note: two or more people cannot sit on the same chair.

–	**Day 2**

4   Determine the number of positive integers $n \leq 1000$ such that the sum of the digits of $5n$ and the sum of the digits of n are the same.

5   Let ABC be a triangle, and K and L be points on AB such that $\angle ACK = \angle KCL = \angle LCB$. Let M be a point in BC such that $\angle MKC = \angle BKM$. If ML is the angle bisector of $\angle KMB$, find $\angle MLC$.

6   A large number of rocks are placed on a table. On each turn, one may remove some rocks from the table following these rules: on the first turn, only one rock may be removed, and on every subsequent turn, one may remove either twice as many rocks or the same number of rocks as they have discarded on the previous turn. Determine the minimum number of turns required to remove exactly 2012 rocks from the table.