Vietnam National Olympiad 1975

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–	Day 1

1   The roots of the equation $x^3 - x + 1 = 0$ are a, b, c. Find $a^8 + b^8 + c^8$.

2   Solve this equation $\frac{y^3+m^3}{(y+m)^3} + \frac{y^3+n^3}{(y+n)^3} + \frac{y^3+p^3}{(y+p)^3} - \frac{3}{2} + \frac{3}{2} \cdot \frac{y-m}{y+m} \cdot \frac{y-n}{y+n} \cdot \frac{y-p}{y+p} = 0$

3   Let $ABCD$ be a tetrahedron with $BA \perp AC, DB \perp (BAC)$. Denote by O the midpoint of AB, and K the foot of the perpendicular from O to DC. Suppose that $AC = BD$. Prove that $\frac{V_{KOAC}}{V_{KOBD}} = \frac{AC}{BD}$ if and only if $2AC \cdot BD = AB^2$.

–	Day 2

4   Find all terms of the arithmetic progression $-1, 18, 37, 56, ...$ whose only digit is 5.

5   Show that the sum of the (local) maximum and minimum values of the function $\frac{tan(3x)}{tan^3 x}$ on the interval $\left(0, \frac{\pi}{2}\right)$ is rational.

6   Let us have a line ℓ in the space and a point A not lying on ℓ. For an arbitrary line ℓ' passing through A, XY (Y is on ℓ') is a common perpendicular to the lines ℓ and ℓ'. Find the locus of points Y.