

Vietnam National Olympiad 1974

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- 1 Find all positive integers n and b with $0 < b < 10$ such that if a_n is the positive integer with n digits, all of them 1, then $a_{2n} - ba_n$ is a square.

- 2
 - i) How many integers n are there such that n is divisible by 9 and $n + 1$ is divisible by 25?
 - ii) How many integers n are there such that n is divisible by 21 and $n + 1$ is divisible by 165?
 - iii) How many integers n are there such that n is divisible by 9, $n + 1$ is divisible by 25, and $n + 2$ is divisible by 4?

- 3 Let ABC be a triangle with $A = 90^\circ$, AH the altitude, P, Q the feet of the perpendiculars from H to AB, AC respectively. Let M be a variable point on the line PQ . The line through M perpendicular to MH meets the lines AB, AC at R, S respectively.
 - i) Prove that circumcircle of ARS always passes the fixed point H .
 - ii) Let M_1 be another position of M with corresponding points R_1, S_1 . Prove that the ratio RR_1/SS_1 is constant.
 - iii) The point K is symmetric to H with respect to M . The line through K perpendicular to the line PQ meets the line RS at D . Prove that $\angle BHR = \angle DHR, \angle DHS = \angle CHS$.

- 4 C is a cube side 1. The 12 lines containing the sides of the cube meet at plane p in 12 points. What can you say about the 12 points?