Vietnam National Olympiad 1976

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–	Day 1
1	Find all integer solutions to $m^{m+n} = n^{12}, n^{m+n} = m^3$.
2	Find all triangles ABC such that $\frac{acosA+bcosB+ccosC}{asinA+bsinB+csinC} = \frac{a+b+c}{9R}$, where, as usual, a, b, c are the lengths of sides BC, CA, AB and R is the circumradius.
3	P is a point inside the triangle ABC. The perpendicular distances from P to the three sides have product p. Show that $p \leq \frac{8S^3}{27abc}$, where $S =$ area ABC and a, b, c are the sides. Prove a similar result for a tetrahedron.
–	Day 2
4	Find all three digit integers $\overline{abc} = n$, such that $\frac{2n}{3} = a!b!c!$
5	L, L' are two skew lines in space and p is a plane not containing either line. M is a variable line parallel to p which meets L at X and L' at Y. Find the position of M which minimises the distance XY. L'' is another fixed line. Find the line M which is also perpendicular to L'' .
6	Show that $\frac{1}{x_1^n} + \frac{1}{x_2^n} + ... + \frac{1}{x_k^n} \geq k^{n+1}$ for positive real numbers x_i with sum 1.