Flanders Math Olympiad 2007

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by Phazz

1 1. The numbers $1, 2, \ldots$ are placed in a triangle as following:

$$\begin{array}{cccc} 1 & & & \\ 2 & 3 & & \\ 4 & 5 & 6 & \\ 7 & 8 & 9 & 10 \\ \ldots & & & \end{array}$$

What is the sum of the numbers on the n-th row?

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2 Given is a half circle with midpoint O and diameter AB. Let Z be a random point inside the half circle, and let X be the intersection of OZ and the half circle, and Y the intersection of AZ and the half circle.

If P is the intersection of BY with the tangent line in X to the half circle, show that $PZ \perp BX$.

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3 Let $ABCD$ be a square with side 10. Let M and N be the midpoints of $[AB]$ and $[BC]$ respectively. Three circles are drawn: one with midpoint D and radius $|AD|$, one with midpoint M and radius $|AM|$, and one with midpoint N and radius $|BN|$. The three circles intersect in the points R, S and T inside the square. Determine the area of $\triangle RST$.

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4 If $f, g : \mathbb{R} \to \mathbb{R}$ are functions that satisfy $f(x + g(y)) = 2x + y \; \forall x, y \in \mathbb{R}$, then determine $g(x + f(y))$.
