Serbia and Montenegro Team Selection Test 2004

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1 Let ABCD be a square and K
be a circle with diameter AB. For an
arbitrary point P on side CD, segments AP and BP meet K
again at
points M and N, respectively, and lines DM and CN meet at point Q.
Prove that Q lies on the circle K
and that AQ : QB = DP : PC.

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2 Let a, b and c be real numbers such that $abc = 1$. Prove that the most two of numbers

$$2a - \frac{1}{b},\ 2b - \frac{1}{c},\ 2c - \frac{1}{a}$$

are greater than 1.

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3 Let $P(x)$ be a polynomial of degree n whose roots are $i-1, i-2, \cdots, i-n$ (where $i^2 = -1$), and let $R(x)$ and $S(x)$ be the polynomials with real coefficients such that $P(x) = R(x) + iS(x)$. Show that the polynomial R has n real roots. (R. Stanojevic)
