Serbia and Montenegro Team Selection Test 2003

www.artofproblemsolving.com/community/c705047

by massnet, elegant

1 If $p(x)$ is a polynomial, denote by $p^n(x)$ the polynomial $p(p(...(p(x))..)$, where p is iterated n times. Prove that the polynomial $p^{2003}(x) - 2p^{2002}(x) + p^{2001}(x)$ is divisible by $p(x) - x$

---

2 Let M and N be the distinct points in the plane of the triangle ABC such that AM : BM : CM = AN : BN : CN. Prove that the line MN contains the circumcenter of ABC.

---

3 Each edge and each diagonal of the convex n-gon $(n \geq 3)$ is colored in red or blue. Prove that the vertices of the n-gon can be labeled as $A_1, A_2, ..., A_n$ in such a way that one of the following two conditions is satisfied:

(a) all segments $A_1A_2, A_2A_3, ..., A_{n-1}A_n, A_nA_1$ are of the same colour.
(b) there exists a number $k, 1 < k < n$ such that the segments $A_1A_2, A_2A_3, ..., A_{k-1}A_k$ are blue, and the segments $A_kA_{k+1}, ..., A_{n-1}A_n, A_nA_1$ are red.
