

ITAMO 2001

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- 1 A hexagon has all its angles equal, and the lengths of four consecutive sides are 5, 3, 6 and 7, respectively. Find the lengths of the remaining two edges.
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- 2 In a basketball tournament every two teams play two matches. As usual, the winner of a match gets 2 points, the loser gets 0, and there are no draws. A single team wins the tournament with 26 points and exactly two teams share the last position with 20 points. How many teams participated in the tournament?
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- 3 Consider the equation

$$x^{2001} = y^x.$$

- Find all pairs (x, y) of solutions where x is a prime number and y is a positive integer.
- Find all pairs (x, y) of solutions where x and y are positive integers.

(Remember that $2001 = 3 \cdot 23 \cdot 29$.)

- 4 A positive integer is called *monotone* if has at least two digits and all its digits are nonzero and appear in a strictly increasing or strictly decreasing order.

(a) Compute the sum of all monotone five-digit numbers.

(b) Find the number of final zeros in the least common multiple of all monotone numbers (with any number of digits).

- 5 Let ABC be a triangle and γ the circle inscribed in ABC . The circle γ is tangent to side AB at the point T . Let D be the point of γ diametrically opposite to T , and S the intersection point of the line through C and D with side AB .

Prove that $AT = SB$.

- 6 A panel contains 100 light bulbs, arranged in a 10 by 10 square array. Some of them are on, the others are off.

The electrical system is such that when the switch corresponding to a light bulb is pressed, all the light bulbs that are on the same row or column of it (including the bulb linked to the pressed switch) change their state (that is they are turned on or off).

- From which starting configurations, pressing the right sequence of switches, is it possible to achieve that all bulbs are on at the same time?

- What is the answer to the previous question if the bulbs are 81, arranged in a 9 by 9 panel?
