**Czech and Slovak Match 1997**

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by parmenides51

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| – | Day 1 |
| 1 | Points $K$ and $L$ are chosen on the sides $AB$ and $AC$ of an equilateral triangle $ABC$ such that $BK = AL$. Segments $BL$ and $CK$ intersect at $P$. Determine the ratio $\frac{AK}{KB}$ for which the segments $AP$ and $CK$ are perpendicular. |
| 2 | In a community of more than six people each member exchanges letters with exactly three other members of the community. Show that the community can be partitioned into two nonempty groups so that each member exchanges letters with at least two members of the group he belongs to. |
| 3 | Find all functions $f : R \to R$ such that $f(f(x)+y) = f(x^2-y)+4f(x)y$ for all $x, y \in R$ . |
| – | Day 2 |
| 4 | Is it possible to place $100$ balls in space so that no two of them have a common interior point and each of them touches at least one third of the others? |
| 5 | The sum of several integers (not necessarily distinct) equals $1492$. Decide whether the sum of their seventh powers can equal (a) $1996$; (b) $1998$. |
| 6 | In a certain language there are only two letters, $A$ and $B$. The words of this language obey the following rules:<br>(i) The only word of length $1$ is $A$;<br>(ii) A sequence of letters $X_1X_2...X_{n+1}$, where $X_i \in \{A, B\}$ for each $i$, forms a word of length $n+1$ if and only if it contains at least one letter $A$ and is not of the form $WA$ for a word $W$ of length $n$.<br>Show that the number of words consisting of $1998 A$s and $1998 B$s and not beginning with $AA$ equals $\binom{3995}{1997} - 1$ |