Czech and Slovak Match 1998

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–	Day 1
1	Let P be an interior point of the parallelogram $ABCD$. Prove that $\angle APB + \angle CPD = 180^\circ$ if and only if $\angle PDC = \angle PBC$.
2	A polynomial $P(x)$ of degree $n \geq 5$ with integer coefficients has n distinct integer roots, one of which is 0. Find all integer roots of the polynomial $P(P(x))$.
3	Let $ABCDEF$ be a convex hexagon such that $AB = BC, CD = DE, EF = FA$. Prove that $\frac{BC}{BE} + \frac{DE}{DA} + \frac{FA}{FC} \geq \frac{3}{2}$. When does equality occur?
–	Day 2
4	Find all functions $f : N \to N - \{1\}$ satisfying $f(n) + f(n+1) = f(n+2) + f(n+3) - 168$ for all $n \in N$.
5	In a triangle ABC, T is the centroid and $\angle TAB = \angle ACT$. Find the maximum possible value of $sin\angle CAT + sin\angle CBT$.
6	In a summer camp there are n girls $D_1, D_2, ..., D_n$ and $2n - 1$ boys $C_1, C_2, ..., C_{2n-1}$. The girl $D_i, i = 1, 2, ..., n$, knows only the boys $C_1, C_2, ..., C_{2i-1}$. Let $A(n, r)$ be the number of different ways in which r girls can dance with r boys forming r pairs, each girl with a boy she knows. Prove that $A(n, r) = \binom{n}{r}\frac{r!}{(n-r)!}$.