Czech and Slovak Match 1999

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–	Day 1
1	Leta,b,c are postive real numbers,proof that $\frac{a}{b+2c}+\frac{b}{c+2a}+\frac{c}{a+2b}\geq 1$
2	The altitudes through the vertices A,B,C of an acute-angled triangle ABC meet the opposite sides at $D,E,F,$ respectively. The line through D parallel to EF meets the lines AC and AB at Q and R, respectively. The line EF meets BC at P. Prove that the circumcircle of the triangle PQR passes through the midpoint of BC.
3	Find all natural numbers k for which there exists a set M of ten real numbers such that there are exactly k pairwise non-congruent triangles whose side lengths are three (not necessarily distinct) elements of M.
–	Day 2
4	Find all positive integers k for which the following assertion holds: If $F(x)$ is polynomial with integer coefficients ehich satisfies $0 \le F(c) \le k$ for all $c\in \{0,1,\cdots,k+1\}$, then $$F(0)=F(1)=\cdots =F(k+1).$$
5	Find all functions $f:(1,\infty)$to R satisfying $f(x)-f(y)=(y-x)f(xy)$ for all $x,y>1$. you may try to find $f(x^5)$ by two ways and then continue the solution. I have also solved by using this method.By finding $f(x^5)$ in two ways I found that $f(x)=xf(x^2)$ for all $x>1$.
6	Prove that for any integer $n\ge3$, the least common multiple of the numbers $1,2,...,n$ is greater than 2^{n-1}.