Singapore Team Selection Test 2006

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– Day 1

1 Let ANC, CLB and BKA be triangles erected on the outside of the triangle ABC such that $\angle NAC = \angle KBA = \angle LCB$ and $\angle NCA = \angle KAB = \angle LBC$. Let D, E, G and H be the midpoints of AB, LK, CA and NA respectively. Prove that $DEGH$ is a parallelogram.

2 Let n be an integer greater than 1 and let $x_1, x_2, ..., x_n$ be real numbers such that $|x_1| + |x_2| + ... + |x_n| = 1$ and $x_1 + x_2 + ... + x_n = 0$
Prove that $\left|\frac{x_1}{1} + \frac{x_2}{2} + \cdots + \frac{x_n}{n}\right| \leq \frac{1}{2}\left(1 - \frac{1}{n}\right)$

3 A pile of n pebbles is placed in a vertical column. This configuration is modified according to the following rules. A pebble can be moved if it is at the top of a column which contains at least two more pebbles than the column immediately to its right. (If there are no pebbles to the right, think of this as a column with 0 pebbles.) At each stage, choose a pebble from among those that can be moved (if there are any) and place it at the top of the column to its right. If no pebbles can be moved, the configuration is called a final configuration. For each n, show that, no matter what choices are made at each stage, the final configuration obtained is unique. Describe that configuration in terms of n.

– Day 2

1 In the plane containing a triangle ABC, points A', B' and C' distinct from the vertices of ABC lie on the lines BC, AC and AB respectively such that AA', BB' and CC' are concurrent at G and $AG/GA' = BG/GB' = CG/GC'$.

Prove that G is the centroid of ABC.

2 Let S be a set of sequences of length 15 formed by using the letters a and b such that every pair of sequences in S differ in at least 3 places. What is the maximum number of sequences in S?

3 Let n be a positive integer such that the sum of all its positive divisors (inclusive of n) equals to $2n + 1$. Prove that n is an odd perfect square.

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