Vietnam Team Selection Test 2016

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– Day 1

1 Find all $a,n\in\mathbb{Z}^+$ $(a>2)$ such that each prime divisor of a^n-1 is also prime divisor of $a^{3^{2016}}-1$

2 Let A be a set contains 2000 distinct integers and B be a set contains 2016 distinct integers. K is the numbers of pairs (m,n) satisfying

$$\begin{cases} m\in A, n\in B\\ |m-n|\le 1000 \end{cases}$$

Find the maximum value of K

3 Let ABC be triangle with circumcircle (O) of fixed BC, $AB\ne AC$ and BC not a diameter. Let I be the incenter of the triangle ABC and $D=AI\cap BC, E=BI\cap CA, F=CI\cap AB$. The circle passing through D and tangent to OA cuts for second time (O) at G ($G\ne A$). GE,GF cut (O) also at M,N respectively.
a) Let $H=BM\cap CN$. Prove that AH goes through a fixed point.
b) Suppose BE,CF cut (O) also at L,K respectively and $AH\cap KL=P$. On EF take Q for $QP=QI$. Let J be a point of the circimcircle of triangle IBC so that $IJ\perp IQ$. Prove that the midpoint of IJ belongs to a fixed circle.

– Day 2

4 Given an acute triangle ABC satisfying $\angle ACB<\angle ABC<\angle ACB+\dfrac{\angle BAC}{2}$. Let D be a point on BC such that $\angle ADC=\angle ACB+\dfrac{\angle BAC}{2}$. Tangent of circumcircle of ABC at A hits BC at E. Bisector of $\angle AEB$ intersects AD and (ADE) at G and F respectively, DF hits AE at H.
a) Prove that circle with diameter AE,DF,GH go through one common point.
b) On the exterior bisector of $\angle BAC$ and ray AC given point K and M respectively satisfying $KB=KD=KM$, On the exterior bisector of $\angle BAC$ and ray AB given point L and N respectively satisfying $LC=LD=LN$. Circle throughs M,N and midpoint I of BC hits BC at P ($P\ne I$). Prove that BM,CN,AP concurrent.

5 Given n numbers $a_1,a_2,...,a_n$ $(n\ge 3)$ where $a_i\in\{0,1\}$ for all $i=1,2.,,,.n$. Consider n following

n-tuples

$$S_1 = (a_1, a_2, ..., a_{n-1}, a_n)$$
$$S_2 = (a_2, a_3, ..., a_n, a_1)$$
$$\vdots$$
$$S_n = (a_n, a_1, ..., a_{n-2}, a_{n-1}).$$

For each tuple $r = (b_1, b_2, ..., b_n)$, let

$$\omega(r) = b_1 \cdot 2^{n-1} + b_2 \cdot 2^{n-2} + \cdots + b_n.$$

Assume that the numbers $\omega(S_1), \omega(S_2), ..., \omega(S_n)$ receive exactly k different values.

a) Prove that $k|n$ and $\frac{2^n-1}{2^k-1} | \omega(S_i) \quad \forall i = 1, 2, ..., n$.

b) Let

$$M = \max_{i=\overline{1,n}} \omega(S_i)$$
$$m = \min_{i=\overline{1,n}} \omega(S_i).$$

Prove that

$$M - m \geq \frac{(2^n-1)(2^{k-1}-1)}{2^k-1}.$$

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6 Given 16 distinct real numbers $\alpha_1, \alpha_2, ..., \alpha_{16}$. For each polynomial P, denote

$$V(P) = P(\alpha_1) + P(\alpha_2) + ... + P(\alpha_{16}).$$

Prove that there is a monic polynomial Q, $\deg Q = 8$ satisfying:

i) $V(QP) = 0$ for all polynomial P has $\deg P < 8$.

ii) Q has 8 real roots (including multiplicity).
