

Puerto Rico Team Selection Test 2014

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by fprosk

- 1 Let $ABCD$ be a parallelogram with $AB > BC$ and $\angle DAB$ less than $\angle ABC$. The perpendicular bisectors of sides AB and BC intersect at the point M lying on the extension of AD . If $\angle MCD = 15^\circ$, find the measure of $\angle ABC$

- 2 We have shortened the usual notation indicating with a sub-index the number of times that a digit is consecutively repeated. For example, 1119900009 is denoted $1_3 9_2 0_4 9_1$.
Find (x, y, z) if $2_x 3_y 5_z + 3_z 5_x 2_y = 5_3 7_2 8_3 5_1 7_3$

- 3 Is it possible to tile an 8×8 board with dominoes (2×1 tiles) so that no two dominoes form a 2×2 square?

- 4 Let S be the set of natural numbers whose digits are different and belong to the set $\{1, 3, 5, 7\}$. Calculate the sum of the elements of S .

- 5 In a cycling competition with 14 stages, one each day, and 100 participants, a competitor was characterized by finishing 93^{rd} each day. What is the best place he could have finished in the overall standings? (Overall standings take into account the total cycling time over all stages.)

- 6 Natural numbers are written in the cells of a 2014×2014 regular square grid such that every number is the average of the numbers in the adjacent cells. Describe and prove how the number distribution in the grid can be.

- 7 Consider N points in the plane such that the area of a triangle formed by any three of the points does not exceed 1. Prove that there is a triangle of area not more than 4 that contains all N points.