

Lusophon Mathematical Olympiad 2017

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– Day 1

- 1 In a math test, there are easy and hard questions. The easy questions worth 3 points and the hard questions worth D points.

If all the questions begin to worth 4 points, the total punctuation of the test increases 16 points.

Instead, if we exchange the questions scores, scoring D points for the easy questions and 3 for the hard ones, the total punctuation of the test is multiplied by $\frac{3}{2}$.

Knowing that the number of easy questions is 9 times bigger the number of hard questions, find the number of questions in this test.

- 2 Let $ABCD$ be a parallelogram, E the midpoint of AD and F the projection of B on CE . Prove that the triangle ABF is isosceles.
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- 3 Determine all the positive integers with more than one digit, all distinct, such that the sum of its digits is equal to the product of its digits.
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– Day 2

- 4 Find how many multiples of 360 are of the form $\overline{ab2017cd}$, where a, b, c, d are digits, with $a \neq 0$.
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- 5 The unit cells of a 5×5 board are painted with 5 colors in a way that every cell is painted by exactly one color and each color is used in 5 cells. Show that exists at least one line or one column of the board in which at least 3 colors were used.
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- 6 Let ABC be a scalene triangle. Consider points D, E, F on segments AB, BC, CA , respectively, such that $\overline{AF} = \overline{DF}$ and $\overline{BE} = \overline{DE}$.
Show that the circumcenter of ABC lies on the circumcircle of CEF .
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