Lusophon Mathematical Olympiad 2014

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– Day 1

1 Four brothers have together forty-eight Kwanzas. If the first brother's money were increased by three Kwanzas, if the second brother's money were decreased by three Kwanzas, if the third brother's money were triplicated and if the last brother's money were reduced by a third, then all brothers would have the same quantity of money. How much money does each brother have?

2 Each white point in the figure below has to be completed with one of the integers $1, 2, ..., 9$, without repetitions, such that the sum of the three numbers in the external circle is equal to the sum of the four numbers in each internal circle that don't belong to the external circle.

(a) Show a solution.

(b) Prove that, in any solution, the number 9 must belong to the external circle.

3 In a convex quadrilateral $ABCD$, P and Q are points on sides BC and DC such that $B\hat{A}P = D\hat{A}Q$. If the line that passes through the orthocenters of $\triangle ABP$ and $\triangle ADQ$ is perpendicular to AC, prove that the area of these triangles are equals.

– Day 2

4 From a point K of a circle, a chord KA (arc AK is greather than 90^o) and a tangent l are drawn. The line that passes through the center of the circle and that is perpendicular to the radius OA, intersects KA at B and l at C. Show that $KC = BC$.

5 Find all quadruples of positive integers (k, a, b, c) such that $2^k = a! + b! + c!$ and $a \geq b \geq c$.

6 Kilua and Ndoti play the following game in a square $ABCD$: Kilua chooses one of the sides of the square and draws a point X at this side. Ndoti chooses one of the other three sides and draws a point Y. Kilua chooses another side that hasn't been chosen and draws a point Z. Finally, Ndoti chooses the last side that hasn't been chosen yet and draws a point W. Each one of the players can draw his point at a vertex of $ABCD$, but they have to choose the side of the square that is going to be used to do that. For example, if Kilua chooses AB, he can draws X at the point B and it doesn't impede Ndoti of choosing BC. A vertex cannot de chosen twice. Kilua wins if the area of the convex quadrilateral formed by X, Y, Z, and W is greater or equal than a half of the area of $ABCD$. Otherwise, Ndoti wins. Which player has a winning strategy? How can he play?