Lusophon Mathematical Olympiad 2012

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– Day 1

1 Arnaldo and Bernaldo train for a marathon along a circular track, which has in its center a mast with a flag raised. Arnaldo runs faster than Bernaldo, so that every 30 minutes of running, while Arnaldo gives 15 laps on the track, Bernaldo can only give 10 complete laps. Arnaldo and Bernaldo left at the same moment of the line and ran with constant velocities, both in the same direction. Between minute 1 and minute 61 of the race, how many times did Arnaldo, Bernaldo and the mast become collinear?

2 Maria has a board of size $n \times n$, initially with all the houses painted white. Maria decides to paint black some houses on the board, forming a mosaic, as shown in the figure below, as follows: she paints black all the houses from the edge of the board, and then leaves white the houses that have not yet been painted. Then she paints the houses on the edge of the next remaining board again black, and so on.
a) Determine a value of n so that the number of black houses equals 200.
b) Determine the smallest value of n so that the number of black houses is greater than 2012.

3 Let n be a positive integer, the players A and B play the following game: we have n balls with the numbers of $1, 2, 3, 4, ..., n$ this balls will be in two boxes with the symbols $\prod$ and $\sum$.
In your turn, the player can choose one ball and the player will put this ball in some box, in the final all the balls of the box $\prod$ are multiplied and we will get a number P, after this all the balls of the box $\sum$ are added up and we will get a number Q(if the box $\prod$ is empty $P = 1$, if the box $\sum$ is empty $Q = 0$).
The player(s) play alternately, player A starts, if $P + Q$ is even player A wins, otherwise player B wins.

a)If $n = 6$, which player has the winning strategy???
b)If $n = 2012$, which player has the winning strategy???

– Day 2

4 An ant decides to walk on the perimeter of an ABC triangle. The ant can start at any vertex. Whenever the ant is in a vertex, it chooses one of the adjacent vertices and walks directly (in a straight line) to the chosen vertex.
a) In how many ways can the ant walk around each vertex exactly twice?
b) In how many ways can the ant walk around each vertex exactly three times?

Note: For each item, consider that the starting vertex is visited.

5   5)Players A and B play the following game: a player writes, in a board, a positive integer n, after this they delete a number in the board and write a new number where can be:
i)The last number p, where the new number will be $p - 2^k$ where k is greatest number such that $p \geq 2^k$
ii) The last number p, where the new number will be $\frac{p}{2}$ if p is even.
The players play alternately, a player win(s) if the new number is equal to 0 and player A starts.

a)Which player has the winning strategy with $n = 40$??
b)Which player has the winning strategy with $n = 2012$??

6   A quadrilateral $ABCD$ is inscribed in a circle of center O. It is known that the diagonals AC and BD are perpendicular. On each side we build semicircles, externally, as shown in the figure.
a) Show that the triangles AOB and COD have the equal areas.
b) If $AC = 8$ cm and $BD = 6$ cm, determine the area of the shaded region.