**Lusophon Mathematical Olympiad 2015**

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| **–** | Day 1 |
| **1** | In a triangle $ABC, L$ and $K$ are the points of intersections of the angle bisectors of $\angle ABC$ and $\angle BAC$ with the segments $AC$ and $BC$, respectively. The segment $KL$ is angle bisector of $\angle AKC$, determine $\angle BAC$. |
| **2** | Determine all ten-digit numbers whose decimal $\overline{a_0a_1a_2a_3a_4a_5a_6a_7a_8a_9}$ is given by such that for each integer $j$ with $0 \leq j \leq 9, a_j$ is equal to the number of digits equal to $j$ in this representation. That is: the first digit is equal to the amount of "$0$" in the writing of that number, the second digit is equal to the amount of "$1$" in the writing of that number, the third digit is equal to the amount of "$2$" in the writing of that number, ... , the tenth digit is equal to the number of "$9$" in the writing of that number. |
| **3** | In the center of a square is a rabbit and at each vertex of this even square, a wolf. The wolves only move along the sides of the square and the rabbit moves freely in the plane. Knowing that the rabbit move at a speed of $10$ km / h and that the wolves move to a maximum speed of $14$ km / h, determine if there is a strategy for the rabbit to leave the square without being caught by the wolves. |
| **–** | Day 2 |
| **4** | Let $a$ be a real number, such that $a \neq 0, a \neq 1, a \neq -1$ and $m, n, p, q$ be natural numbers . Prove that if $a^m + a^n = a^p + a^q$ and $a^{3m} + a^{3n} = a^{3p} + a^{3q}$ , then $m \cdot n = p \cdot q$. |
| **5** | Two circles of radius $R$ and $r$, with $R > r$, are tangent to each other externally. The sides adjacent to the base of an isosceles triangle are common tangents to these circles. The base of the triangle is tangent to the circle of the greater radius. Determine the length of the base of the triangle. |
| **6** | Let $(a_n)$ be defined by: $$a_1 = 2, \qquad a_{n+1} = a_n^3 - a_n + 1$$ Consider positive integers $n, p$, where $p$ is an odd prime. Prove that if $p \mid a_n$, then $p > n$. |