Lusophon Mathematical Olympiad 2016

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by parmenides51, mathisreal, Mathematics73

– Day 1

1 Consider 10 distinct positive integers that are all prime to each other (that is, there is no a prime factor common to all), but such that any two of them are not prime to each other. What is the smallest number of distinct prime factors that may appear in the product of 10 numbers?

2 The circle ω_1 intersects the circle ω_2 in the points A and B, a tangent line to this circles intersects ω_1 and ω_2 in the points E and F respectively. Suppose that A is inside of the triangle BEF, let H be the orthocenter of BEF and M is the midpoint of BH. Prove that the centers of the circles ω_1 and ω_2 and the point M are collinears.

3 Suppose a real number a is a root of a polynomial with integer coefficients $P(x) = a_nx^n + a_{n-1}x^{n-1} + ... + a_1x + a_0$.
Let $G = |a_n| + |a_{n-1}| + ... + |a_1| + |a_0|$. We say that G is a *gingado* of a.
For example, as 2 is root of $P(x) = x^2 + x - 2, G = |1| + |1| + |-2| = 4$, we say that 4 is a *gingado* of 2.
What is the fourth largest real number a such that 3 is a *gingado* of a?

– Day 2

4 8 CPLP football teams competed in a championship in which each team played one and only time with each of the other teams. In football, each win is worth 3 points, each draw is worth 1 point and the defeated team does not score. In that championship four teams were in first place with 15 points and the others four came in second with N points each. Knowing that there were 12 draws throughout the championship, determine N.

5 A numerical sequence is called lusophone if it satisfies the following three conditions:
i) The first term of the sequence is number 1.
ii) To obtain the next term of the sequence we can multiply the previous term by a positive prime number $(2, 3, 5, 7, 11, ...)$ or add 1.
(iii) The last term of the sequence is the number 2016.
For example: $1 \overset{\times 11}{\to} 11 \overset{\times 61}{\to} 671 \overset{+1}{\to} 672 \overset{\times 3}{\to} 2016$
How many Lusophone sequences exist in which (as in the example above) the add 1 operation was used exactly once and not multiplied twice by the same prime number?

6 Source: Lusophon MO 2016

Prove that any positive power of 2 can be written as:

$$5xy - x^2 - 2y^2$$

where x and y are odd numbers.