

Argentina Cono Sur Team Selection Test 2013

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– **Day 1**

1 2000 people are standing on a line. Each one of them is either a *liar*, who will always lie, or a *truth-teller*, who will always tell the truth. Each one of them says: "there are more liars to my left than truth-tellers to my right". Determine, if possible, how many people from each class are on the line.

2 If $x \neq 1, y \neq 1, x \neq y$ and

$$\frac{yz - x^2}{1 - x} = \frac{xz - y^2}{1 - y}$$

show that both fractions are equal to $x + y + z$.

3 1390 ants are placed near a line, such that the distance between their heads and the line is less than 1cm and the distance between the heads of two ants is always larger than 2cm. Show that there is at least one pair of ants such that the distance between their heads is at least 10 meters (consider the head of an ant as point).

– **Day 2**

4 Show that the number $N = \underbrace{44\dots4}_n \underbrace{88\dots8}_n - 1 \underbrace{33\dots3}_{n-1} 2$ is a perfect square for all positive integers n .

5 Let ABC be an equilateral triangle and D a point on side AC . Let E be a point on BC such that $DE \perp BC$, F on AB such that $EF \perp AB$, and G on AC such that $FG \perp AC$. Lines FG and DE intersect in P . If M is the midpoint of BC , show that BP bisects AM .

6 Let $m \geq 4$ and $n \geq 4$. An integer is written on each cell of a $m \times n$ board. If each cell has a number equal to the arithmetic mean of some pair of numbers written on its neighbouring cells, determine the maximum amount of distinct numbers that the board may have.

Note: two neighbouring cells share a common side.