Argentina Cono Sur Team Selection Test 2014

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–	**Day 1**
1	A positive integer N is written on a board. In a step, the last digit c of the number on the board is erased, and after this, the remaining number m is erased and replaced with $\|m-3c\|$ (for example, if the number 1204 is on the board, after one step, we will replace it with $120-3\cdot 4=108$). We repeat this until the number on the board has only one digit. Find all positive integers N such that after a finite number of steps, the remaining one-digit number is 0.
2	The numbers 1 through 9 are written on a 3×3 board, without repetitions. A valid operation is to choose a row or a column of the board, and replace its three numbers a, b, c (in order, i.e., the first number of the row/column is a, the second number of the row/column is b, the third number of the row/column is c) with either the three non-negative numbers $a-x, b-x, c+x$ (in order) or with the three non-negative numbers $a+x, b-x, c-x$ (in order), where x is a real positive number which may vary in each operation . a) Determine if there is a way of getting all 9 numbers on the board to be the same, starting with the following board: $\begin{array}{\|c\|c\|c\|}\hline 1 & 2 & 3 \\ \hline 4 & 5 & 6 \\ \hline 7 & 8 & 9 \\ \hline \end{array}$ b) For all posible configurations such that it is possible to get all 9 numbers to be equal to a number m using the valid operations, determine the maximum value of m.
3	All diagonals of a convex pentagon are drawn, dividing it in one smaller pentagon and 10 triangles. Find the maximum number of triangles with the same area that may exist in the division.
–	**Day 2**
4	Find all pairs of positive prime numbers (p, q) such that $p^5+p^3+2=q^2-q$
5	In an acute triangle ABC, let D be a point in BC such that AD is the angle bisector of $\angle BAC$. Let $E\neq B$ be the point of intersection of the circumcircle of triangle ABD with the line perpendicular to AD drawn through B. Let O be the circumcenter of triangle ABC. Prove that E,

O, and A are collinear.

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6 120 bags with 100 coins are placed on the floor. One bag has coins that weigh 9 grams, the other bags have coins that weigh 10 grams. One may place some coins (not necessarily from the same bag) on a weighing scale, but it will only properly display the weight if it is less than 1000 grams. Determine the minimum number of times that the weighing scale may be used in order to identify the bag that has the 9-gram coins.
