Rioplatense Mathematical Olympiad, Level 3 2017

www.artofproblemsolving.com/community/c712938

by parmenides51

–	Day 1
1	Let a be a fixed positive integer. Find the largest integer b such that $(x+a)(x+b)=x+a+b$, for some integer x.
2	One have n distinct circles(with the same radius) such that for any $k+1$ circles there are (at least) two circles that intersects in two points. Show that for each line l one can make k lines, each one parallel with l, such that each circle has (at least) one point of intersection with some of these lines.
3	Show that there are infinitely many pairs of positive integers (m,n), with $m<n$, such that m divides $n^{2016}+n^{2015}+\cdots+n^2+n+1$ and n divides $m^{2016}+m^{2015}+\cdots+m^2+m+1$.
–	Day 2
4	Is there a number n such that one can write n as the sum of 2017 perfect squares and (with at least) 2017 distinct ways?
5	Let ABC be a triangle and I is your incenter, let P be a point in AC such that PI is perpendicular to AC, and let D be the reflection of B wrt circumcenter of $\triangle ABC$. The line DI intersects again the circumcircle of $\triangle ABC$ in the point Q. Prove that QP is the angle bisector of the angle $\angle AQC$.
6	For each fixed positiver integer n, $n\geq 4$ and P an integer, let $(P)_n\in[1,n]$ be the smallest positive residue of P modulo n. Two sequences $a_1,a_2,\ldots,a_k$ and $b_1,b_2,\ldots,b_k$ with the terms in $[1,n]$ are defined as equivalent, if there is t positive integer, gcd$(t,n)=1$, such that the sequence $(ta_1)_n,\ldots,(ta_k)_n$ is a permutation of $b_1,b_2,\ldots,b_k$. Let α a sequence of size n and your terms are in $[1,n]$, such that each term appears h times in the sequence α and $2h\geq n$. Show that α is equivalent to some sequence β which contains a subsequence such that your size is(at most) equal to h and your sum is exactly equal to n.

© 2022 AoPS Incorporated

Art of Problem Solving is an ACS WASC Accredited School.