Rioplatense Mathematical Olympiad, Level 3 2012

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– Day 1

1 An integer n is called *apocalyptic* if the addition of 6 different positive divisors of n gives 3528. For example, 2012 is apocalyptic, because it has six divisors, 1, 2, 4, 503, 1006 and 2012, that add up to 3528.

Find the smallest positive apocalyptic number.

2 A rectangle is divided into n^2 smaller rectangle by $n-1$ horizontal lines and $n-1$ vertical lines. Between those rectangles there are exactly 5660 which are not congruent. For what minimum value of n is this possible?

3 Let T be a non-isosceles triangle and $n \geq 4$ an integer . Prove that you can divide T in n triangles and draw in each of them an inner bisector so that those n bisectors are parallel.

– Day 2

4 Find all real numbers x, such that:
a) $\lfloor x \rfloor + \lfloor 2x \rfloor + ... + \lfloor 2012x \rfloor = 2013$
b) $\lfloor x \rfloor + \lfloor 2x \rfloor + ... + \lfloor 2013x \rfloor = 2014$

5 Let $a \geq 2$ and $n \geq 3$ be integers . Prove that one of the numbers $a^n + 1, a^{n+1} + 1, ..., a^{2n-2} + 1$ does not share any odd divisor greater than 1 with any of the other numbers.

6 In each square of a 100×100 board there is written an integer. The allowed operation is to choose four squares that form the figure or any of its reflections or rotations, and add 1 to each of the four numbers. The aim is, through operations allowed, achieving a board with the smallest possible number of different residues modulo 33. What is the minimum number that can be achieved with certainty?