Rioplatense Mathematical Olympiad, Level 3 2015

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– Day 1

1 Let ABC be a triangle and P a point on the side BC. Let S_1 be the circumference with center B and radius BP that cuts the side AB at D such that D lies between A and B. Let S_2 be the circumference with center C and radius CP that cuts the side AC at E such that E lies between A and C. Line AP cuts S_1 and S_2 at X and Y different from P, respectively. We call T the point of intersection of DX and EY. Prove that $\angle BAC + 2\angle DTE = 180$

2 Let a, b, c positive integers, coprime. For each whole number $n \ge 1$, we denote by $s(n)$ the number of elements in the set $\{a, b, c\}$ that divide n. We consider $k_1 < k_2 < k_3 <$ ... .the sequence of all positive integers that are divisible by some element of $\{a, b, c\}$. Finally we define the characteristic sequence of (a, b, c) like the succession $s(k_1), s(k_2), s(k_3), ...$.
Prove that if the characteristic sequences of (a, b, c) and (a', b', c') are equal, then $a = a', b = b'$ and $c = c'$

3 We say an integer number $n \ge 1$ is conservative, if the smallest prime divisor of $(n!)^n + 1$ is at most $n + 2015$. Decide if the number of conservative numbers is infinite or not.

– Day 2

4 You have a 9×9 board with white squares. A tile can be moved from one square to another neighbor (tiles that share one side). If we paint some squares of black, we say that such coloration is *good* if there is a white square where we can place a chip that moving through white squares can return to the initial square having passed through at least 3 boxes, without passing the same square twice.
Find the highest possible value of k such that any form of painting k squares of black are a *good* coloring.

5 For a positive integer number n we denote $d(n)$ as the greatest common divisor of the binomial coefficients $\binom{n+1}{n}, \binom{n+2}{n}, ..., \binom{2n}{n}$.
Find all possible values of $d(n)$

6 Let ABC be an acut-angles triangle of incenter I, circumcenter O and inradius r. Let ω be the inscribed circle of the triangle ABC. A_1 is the point of ω such that AIA_1O is a convex trapezoid of bases AO and IA_1. Let ω_1 be the circle of radius r which goes through A_1, tangent to the line AB and is different from ω . Let ω_2 be the circle of radius r which goes through A_1, is tangent to the line AC and is different from ω . Circumferences ω_1 and ω_2 they are cut at points A_1 and

A_2. Similarly are defined points B_2 and C_2. Prove that the lines AA_2, BB_2 and $CC2$ they are concurrent.