Rioplatense Mathematical Olympiad, Level 3 2007

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– Day 1

1 Determine the values of $n \in N$ such that a square of side n can be split into a square of side 1 and five rectangles whose side measures are 10 distinct natural numbers and all greater than 1.

2 Let ABC be a triangle with incenter I . The circle of center I which passes through B intersects AC at points E and F, with E and F between A and C and different from each other. The circle circumscribed to triangle IEF intersects segments EB and FB at Q and R, respectively. Line QR intersects the sides AB and BC at P and S, respectively.
If a, b and c are the measures of the sides BC, CA and AB, respectively, calculate the measurements of BP and BS.

3 Let $p > 3$ be a prime number and x an integer, denote by $r(x) \in \{0, 1, ..., p-1\}$ to the rest of x modulo p . Let $x_1, x_2, ..., x_k$ ($2 < k < p$) different integers modulo p and not divisible by p.
We say that a number $a \in \{1, 2, ..., p-1\}$ is *good* if $r(ax_1) < r(ax_2) < ... < r(ax_k)$.
Show that there are at most $\frac{2p}{k+1} - 1$ *good* numbers.

– Day 2

4 Find all functions $f : Z \to Z$ with the following property: if $x+y+z = 0$, then $f(x)+f(y)+f(z) = xyz$.

5 Divide each side of a triangle into 50 equal parts, and each point of the division is joined to the opposite vertex by a segment. Calculate the number of intersection points determined by these segments.

Clarification : the vertices of the original triangle are not considered points of intersection or division.

6 Let $n > 2$ be a natural number. A subset A of R is said n-*small* if there exist n real numbers $t_1, t_2, ..., t_n$ such that the sets $t_1 + A, t_2 + A, ..., t_n + A$ are different . Show that R can not be represented as a union of $n-1$ n-*small* sets .

Notation : if $r \in R$ and $B \subset R$, then $r + B = \{r + b | b \in B\}$.