Cono Sur Olympiad 1996

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– Day 1

1 In the following figure, the largest square is divided into two squares and three rectangles, as shown:

The area of each smaller square is equal to a and the area of each small rectangle is equal to b. If $a+b=24$ and the root square of a is a natural number, find all possible values for the area of the largest square.
`https://cdn.artofproblemsolving.com/attachments/f/a/0b424d9c293889b24d9f31b1531bed5081081 png`

2 Consider a sequence of real numbers defined by:

$a_{n+1} = a_n + \frac{1}{a_n}$ for $n = 0, 1, 2, ...$

Prove that, for any positive real number a_0, is true that a_{1996} is greater than 63.

3 A shop sells bottles with this capacity: $1L, 2L, 3L, ..., 1996L$, the prices of bottles satifies this 2 conditions: $1.$ Two bottles have the same price, if and only if, your capacities satifies $m-n=1000$ $2.$ The price of bottle $m(1001>m>0)$ is $1996-m$ dollars.
Find all pair(s) m and n such that:
a) $m+n=1000$
b) the cost is smallest possible!!!
c) with the pair, the shop can measure k liters, with $0<k<1996$(for all k integer)
Note: The operations to measure are:
i) To fill or empty any one of two bottles
ii)Pass water of a bottle for other bottle
We can measure k liters when the capacity of one bottle plus the capacity of other bottle is equal to k

– Day 2

4 The sequence $0, 1, 1, 1, 1, 1, ..., 1$ where have 1 number zero and 1995 numbers one.
If we choose two or more numbers in this sequence(but not the all 1996 numbers) and substitute one number by arithmetic mean of the numbers selected, we obtain a new sequence with 1996 numbers!!!
Show that, we can repeat this operation until we have all 1996 numbers are equal
Note: It's not necessary to choose the same quantity of numbers in each operation!!!

5 We want to cover totally a square(side is equal to k integer and $k > 1$) with this rectangles: 1 rectangle (1×1), 2 rectangles (2×1), 4 rectangles (3×1),...., 2^n rectangles $(n + 1 \times 1)$, such that the rectangles can't overlap and don't exceed the limits of square.
Find all k, such that this is possible and for each k found you have to draw a solution

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6 Find all integers $n \le 3$ such that there is a set S_n formed by n points of the plane that satisfy the following two conditions:
Any three points are not collinear.
No point is found inside the circle whose diameter has ends at any two points of S_n.

NOTE: The points on the circumference are not considered to be inside the circle.
