Cono Sur Olympiad 1997

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–	Day 1
1	We have 98 cards, in each one we will write one of the numbers: $1, 2, 3, 4, ..., 97, 98$. We can order the 98 cards, in a sequence such that two consecutive numbers X and Y and the number $X - Y$ is greater than 48, determine how and how many ways we can make this sequence!!
2	Let C be a circunference, O is your circumcenter, AB is your diameter and R is any point in C (R is different of A and B) Let P be the foot of perpendicular by O to AR, in the line OP we match a point Q, where QP is $\frac{OP}{2}$ and the point Q isn't in the segment OP. In Q, we will do a parallel line to AB that cut the line AR in T. Denote H the point of intersections of the line AQ and OT. Show that H, B and R are collinears.
3	Show that, exist infinite triples (a, b, c) where a, b, c are natural numbers, such that: $2a^2 + 3b^2 - 5c^2 = 1997$
–	Day 2
4	Consider a board with n rows and 4 columns. In the first line are written 4 zeros (one in each house). Next, each line is then obtained from the previous line by performing the following operation: one of the houses, (that you can choose), is maintained as in the previous line; the other three are changed: * if in the previous line there was a 0, then in the down square 1 is placed; * if in the previous line there was a 1, then in the down square 2 is placed; * if in the previous line there was a 2, then in the down square 0 is placed; Build the largest possible board with all its distinct lines and demonstrate that it is impossible to build a larger board.
5	Let n be a natural number $n > 3$. Show that in the multiples of 9 less than 10^n, exist more numbers with the sum of your digits equal to $9(n - 2)$ than numbers with the sum of your digits equal to $9(n - 1)$.
6	Let ABC be a acute-angle triangle and X be point in the plane of this triangle. Let M, N, P be the orthogonal projections of X in the lines that contains the altitudes of this triangle

Determine the positions of the point X such that the triangle MNP is congruent to ABC