

Cono Sur Olympiad 1998

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– Day 1

- 1** We have 98 cards, in each one we will write one of the numbers: 1, 2, 3, 4, ..., 97, 98. We can order the 98 cards, in a sequence such that two consecutive numbers X and Y and the number $X - Y$ is greater than 48, determine how and how many ways we can make this sequence!!
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- 2** Let H be the orthocenter of the triangle ABC , M is the midpoint of the segment BC . Let X be the point of the intersection of the line HM with arc BC (without A) of the circumcircle of ABC , let Y be the point of intersection of the line BH with the circle, show that $XY = BC$.
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- 3** Prove that, least 30
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– Day 2

- 4** Find all functions $R \rightarrow R$ such that: $f(x^2) - f(y^2) + 2x + 1 = f(x + y)f(x - y)$
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- 5** In *Terra Brasilis* there are n houses where n goblins live, each in a house. There are one-way routes such that:
- each route joins two houses,
 - in each house exactly one route begins,
 - in each house exactly one route ends.
- If a route goes from house A to house B , then we will say that house B is next to house A . This relationship is not symmetric, that is: in this situation, not necessarily house A is next to house B .
- Every day, from day 1, each goblin leaves the house where he is and arrives at the next house. A legend of *Terra Brasilis* says that when all the goblins return to the original position, the world will end.
- a) Show that the world will end.
b) If $n = 98$, show that it is possible for elves to build and guide the routes so that the world does not end before 300,000 years.
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- 6** The mayor of a city wishes to establish a transport system with at least one bus line, in which:
- each line passes exactly three stops,
 - every two different lines have exactly one stop in common,
 - for each two different bus stops there is exactly one line that passes through both.
- Determine the number of bus stops in the city.
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