Cono Sur Olympiad 1999

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by parmenides51

–	Day 1	
1	Find the smallest positive integer n such that the 73 fractions $\frac{19}{n+21}, \frac{20}{n+22}, \frac{21}{n+23}, ..., \frac{91}{n+93}$ are all irreducible.	
2	Let ABC be a triangle right in A. Construct a point P on the hypotenuse BC such that if Q is the foor of the perpendicular drawn from P to side AC, then the area of the square of side PQ is equal to the area of the rectangle of sides PB and PC. Show construction steps.	
3	There are 1999 balls in a row, some are red and some are blue (it could be all red or all blue). Under every ball we write a number equal to the sum of the amount of red balls in the right of this ball plus the sum of the amount of the blue balls that are in the left of this ball. In the sequence of numbers that we get with this balls we have exactly three numbers that appears an odd number of times, which numbers could these three be?	
–	Day 2	
4	Let A be a six-digit number, three of which are colored and equal to $1, 2$, and 4. Prove that it is always possible to obtain a number that is a multiple of 7, by performing only one of the following operations: either delete the three colored figures, or write all the numbers of A in some order.	
5	Give a square of side 1. Show that for each finite set of points of the sides of the square you can find a vertex of the square with the following property: the arithmetic mean of the squares of the distances from this vertex to the points of the set is greater than or equal to $3/4$.	
6	An ant walks across the floor of a circular path of radius r and moves in a straight line, but sometimes stops. Each time it stops, before resuming the march, it rotates 60^o alternating the direction (if the last time it turned 60^o to its right, the next one does it 60^o to its left, and vice versa). Find the maximum possible length of the path the ant goes through. Prove that the length found is, in fact, as long as possible. Figure: turn 60^o to the right .	