Silk Road Mathematics Competiton 2003

www.artofproblemsolving.com/community/c714757

by Ovchinnikov Denis

1 Let $a_1, a_2, ..., a_{2003}$ be sequence of reals number.
Call a_k $leading$ element, if at least one of expression $a_k; a_k + a_{k+1}; a_k + a_{k+1} + a_{k+2}; ....; a_k + ak + 1 + a_{k+2} + ... + a_{2003}$ is positive.
Prove, that if exist at least one $leading$ element, then sum of all $leading$'s elements is positive.

Official solution here (`http://www.artofproblemsolving.com/Forum/viewtopic.php?f=125\&t=365714\&p=201165\#p2011659`)

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2 Let $s = \frac{AB+BC+AC}{2}$ be half-perimeter of triangle ABC. Let L and Nbe a point's on ray's AB and CB, for which $AL = CN = s$. Let K is point, symmetric of point B by circumcenter of ABC. Prove, that perpendicular from K to NL passes through incenter of ABC.

Solution for problem here (`http://www.artofproblemsolving.com/Forum/viewtopic.php?f=125\&t=365714\&p=201165\#p2011659`)

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3 Let $0 < a < b < 1$ be reals numbers and

$$g(x) = \begin{cases} x+1-a, & \text{if } 0 < x < b \\ b-a, & \text{if } x = a \\ x-a, & \text{if } a < x < b \\ 1-a, & \text{if } x = b \\ x-a, & \text{if } b < x < 1 \end{cases}$$

Give that there exist $n+1$ reals numbers $0 < x_0 < x_1 < ... < x_n < 1$, for which $g^{[n]}(x_i) = x_i$ $(0 \leq i \leq n)$. Prove that there exists a positive integer N, such that $g^{[N]}(x) = x$ for all $0 < x < 1$.

$(g^{[n]}(x) = \underbrace{g(g(....(g(x))....))}_{\text{n times}})$

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4 Find $\sum_{k \in A} \frac{1}{k-1}$ where $A = \{m^n : m, n \in \mathbb{Z} m, n \geq 2\}$.

Problem was post earlier here (`http://www.artofproblemsolving.com/Forum/viewtopic.php?f=67\&t=29456\&hilit=silk+road`) , but solution not gives and olympiad doesn't indicate, so I post it again :blush:

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