

Silk Road Mathematics Competiton 2004

www.artofproblemsolving.com/community/c714770

by Ovchinnikov Denis

- 1 Find all $f : \mathbb{R} \rightarrow \mathbb{R}$, such that $(x + y)(f(x) - f(y)) = (x - y)f(x + y)$ for all real x, y .

- 2 find all primes p , for which exist natural numbers, such that $p = m^2 + n^2$ and $p | (m^3 + n^3 - 4)$.

- 3 In-circle of ABC with center I touch AB and AC at P and Q respectively. BI and CI intersect PQ at K and L respectively. Prove, that circumcircle of ILK touch incircle of ABC iff $|AB| + |AC| = 3|BC|$.

- 4 Natural $n \geq 2$ is given. Group of people calls $n - compact$, if for any men from group, we can found n people (without he), each two of there are familiar.
Find maximum N such that for any $n - compact$ group, consisting N people contains subgroup from $n + 1$ people, each of two of there are familiar.
