Silk Road Mathematics Competiton 2005

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1 Let $n \geq 2$ be natural number.
Prove, that $(1^{n-1}+2^{n-1}+....+(n-1)^{n-1})+1$ divided by n iff for any prime divisor p of n $p|\frac{n}{p}-1$ and $(p-1)|\frac{n}{p}-1$.

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2 Find all $(m,n) \in \mathbb{Z}^2$ that we can color each unit square of $m \times n$ with the colors black and white that for each unit square number of unit squares that have the same color with it and have at least one common vertex (including itself) is even.

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3 Assume A,B,C are three collinear points that $B \in [AC]$. Suppose AA' and BB' are to parralel lines that A', B' and C are not collinear. Suppose O_1 is circumcenter of circle passing through A, A' and C. Also O_2 is circumcenter of circle passing through B, B' and C. If area of $A'CB'$ is equal to area of O_1CO_2, then find all possible values for $\angle CAA'$

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4 Suppose $\{a(n)\}_{n=1}^\infty$ is a sequence that:

$$a(n)=a(a(n-1))+a(n-a(n-1)) \quad \forall\, n \geq 3$$

and $a(1)=a(2)=1$.

Prove that for each $n \geq 1$, $a(2n) \leq 2a(n)$.
