Silk Road Mathematics Competiton 2002

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1 Let $\triangle ABC$ be a triangle with incircle $\omega(I,r)$and circumcircle $\zeta(O,R)$.Let l_a be the angle bisector of $\angle BAC$.Denote $P=l_a\cap\zeta$.Let D be the point of tangency ω with $[BC]$.Denote $Q=PD\cap\zeta$.Show that $PI=QI$ if $PD=r$.

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2 I tried to search SRMC problems,but i didn't find them(I found only SRMC 2006).Maybe someone know where on this site i could find SRMC problems?I have all SRMC problems,if someone want i could post them, :wink:

Here is one of them,this is one nice inequality from first SRMC:

Let n be an integer with $n>2$ and $a_1,a_2,\dots,a_n\in R^+$.Given any positive integers t,k,p with $1<t<n$,set $m=k+p$,prove the following inequalities:

a) $\frac{a_1^p}{a_2^k+a_3^k+\dots+a_t^k}+\frac{a_2^p}{a_3^k+a_4^k+\dots+a_{t+1}^k}+\dots+\frac{a_{n-1}^p}{a_n^k+a_1^k+\dots+a_{t-2}^k}+\frac{a_n^p}{a_1^k+a_2^k+\dots+a_{t-1}^k}\geq\frac{(a_1^p+a_2^p\dots+a_n^p)^2}{(t-1)(a_1^m+a_2^m+\dots+a_n^m)}$

b)$\frac{a_2^k+a_3^k\dots+a_t^k}{a_1^p}+\frac{a_3^k+a_4^k\dots+a_{t+1}^k}{a_2^p}+\dots+\frac{a_1^k+a_2^k\dots+a_{t-1}^k}{a_n^p}\geq\frac{(t-1)(a_1^k+a_2^k\dots+a_n^k)^2}{(a_1^m+a_2^m+\dots+a_n^m)}$:wink:

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3 In each unit cell of a finite set of cells of an infinite checkered board, an integer is written so that the sum of the numbers in each row, as well as in each column, is divided by 2002. Prove that every number α can be replaced by a certain number α' , divisible by 2002 so that $|\alpha-\alpha'|<2002$ and the sum of the numbers in all rows, and in all columns will not change.

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4 Observe that the fraction $\frac{1}{7}=0,142857$ is a pure periodical decimal with period $6=7-1$,and in one period one has $142+857=999$.For $n=1,2,\dots$ find a sufficient and necessary condition that the fraction $\frac{1}{2n+1}$ has the same properties as above and find two such fractions other than $\frac{1}{7}$.
