Silk Road Mathematics Competiton 2007

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1 On the board are written $2, 3, 5, ..., 2003$, that is, all the prime numbers of the interval $[2, 2007]$. The operation of *simplification* is the replacement of two numbers a, b by a maximal prime number not exceeding $\sqrt{a^2 - ab + b^2}$. First, the student erases the number $q, 2 < q < 2003$, then applies the *simplification* operation to the remaining numbers until one number remains. Find the maximum possible and minimum possible values of the number obtained in the end. How do these values depend on the number q?

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2 Let ω be the incircle of triangle ABC touches BC at point K . Draw a circle passing through points B and C , and touching ω at the point S . Prove that SK passes through the center of the exscribed circle of triangle ABC , tangent to side BC .

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3 Find the max. value of M,such that for all $a,b,c>0$:
$a^{3}+b^{3}+c^{3}-3abc\geq M(|a-b|^{3}+|a-c|^{3}+|c-b|^{3})$

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4 The set of polynomials $f_1, f_2, \ldots, f_n$ with real coefficients is called *special* , if for any different $i, j, k \in \{1, 2, \ldots, n\}$ polynomial $\frac{2}{3} f_i + f_j + f_k$ has no real roots, but for any different $p, q, r, s \in \{1, 2, \ldots, n\}$ of a polynomial $f_p + f_q + f_r + f_s$ there is a real root.
a) Give an example of a *special* set of four polynomials whose sum is not a zero polynomial.
b) Is there a *special* set of five polynomials?
